Convergency and Regulation

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1. Convergency

In many control problems such as the output regulation problem it is required that controllers be designed in such a way that all the solutions of the corresponding closed loop system 'forget' their initial conditions and converge to some steady state solution, which is only determined only by the input of the closed loop system. This input can be, for example, a command signal or a signal generated by a feed-forward part of the controller or as in the output regulation problem, it can be the signal generated by the exo-system. For asymptotically stable linear systems excited by inputs, this is a natural property. Indeed, due to the linearity of the system, every solution is globally asymptotically stable and therefore, all solutions of such a system 'forget' their initial conditions and converge to each other. After the transients have died out, the dynamics of the system are solely determined by the input. For nonlinear control systems in general global asymptotic stability of a system with zero input does not guarantee that all solutions of this system with a nonzero input 'forget' their initial conditions and converge to each other. This implies that for nonlinear systems the convergent dynamics property requires additional conditions. 'Forgetting' the initial conditions has been studied in a number of papers, most notably in the Russian literature V.A. Pliss, [1], and afterwards B.P. Demidovich, [2], have introduced the notion of convergent systems. In the context of a control system, the definition of a convergent system requires that there exists, for each (bounded) input function, a unique globally asymptotically stable solution that is bounded over the whole time axis. Other relevant contributions on similar notions of stability can be found in [3], [4], and in more recent times [5], [6], [7] and [8]. A concise review of the work of Demidovich, together with a sufficient condition for convergency is presented in [9].

2. Regulation

Regulation or rather output regulation serves as a central problem in control, and deals with the asymptotic tracking of prescribed reference signals and/or the asymptotic rejection of undesired disturbances in the output of a system. As such, many control problems can be cast in the frame of an output regulation problem. The linear output regulation problem was completely solved in the 1970s by B.A. Francis, W.M. Wonham, E.J. Davison and others, resulting in the famous 'internal model principle' in combination with the ubiquitous 'regulator equations', see e.g. [10]. Several authors have addressed the nonlinear output regulation problem, with, amongst others, the seminal paper of C.I. Byrnes and A. Isidori, [11], describing the solvability in terms of the nonlinear regulator equations. Most of the work on the output regulator problem, and variations thereof, has concentrated on local or approximate versions of the problem, mostly due to the inherent difficulty to solve the regulator equations. Extensions of the problem to the global or semi-global case have only more recently been addressed, e.g. in [12]. By using the notion of convergency, a slightly different, but in some sense, more natural formulation of the output regulation problem is described. In particular, it is required that the solution of the regulation problem is such that the closed loop dynamics is convergent, so that a natural and unique steady state solution exists in all cases, and the initial conditions do not appear any longer in this solution. A detailed treatise on this approach to the output regulation problem is given in [13].

3. Frequency Response Functions

Convergent systems have the appealing property that periodic input functions give rise to periodic steady-state solutions, thus allowing for the particular discussion of what the steady state response is to harmonic input signals. In a linear context this information is usually captured in the frequency response plot or Bode plot describing both the amplitude and phase shift of the response as a function of the input amplitude and frequency. The (amplitude) frequency response function thus can be introduced for convergent control systems, and it forms a natural measure for which frequencies the system amplifies harmonic input signals. Although this is still far from a complete nonlinear frequency domain thinking, this approach serves as a first step towards a frequency domain based approach to controller design, in the sense that one may ask the closed loop dynamics to have specific amplitude shaping properties in certain frequency ranges. Obviously, this needs to be developed further since no analogy of 'phase' seems available in this manner. The subject of convergent dynamics based frequency response functions can be found in [13].

4. Conclusions

The notion of convergent dynamics provides a useful and valuable tool for studying nonlinear control systems. Control systems that belong to the class of convergent systems have a number of appealing properties, which resemble those of linear control systems. One could argue for this reason that convergent systems are a natural extension of linear systems.

References

[1] V.A. Pliss, *Nonlocal problems of the theory of oscillations*, Academic Press, London (1966).

[2] B.P. Demidovich, *Lectures on stability theory* (in Russian), Nauka, Moscow, (1967).

[3] T. Yoshizawa, *Stability theory and the existence of periodic and almost periodic solutions*, Springer Verlag, New York, (1975).

[4] J.P. LaSalle and S. Lefschetz, *Stability of Liapunov's direct method with applications*, Academic Press, New York, (1961).

[5] V. Fromion, S. Monaco and D. Normand-Cyrot, *A link between input-output stability and Lyapunov stability*, Systems and Control Letters, **27**, pp.243-248, (1996).

[6] W. Lohmiller and J.-J.E. Slotine, *On contraction analysis for nonlinear systems*, Automatica **34**, pp. 683-696, (1998).

[7] D. Angeli, *A Lyapunov approach to incremental stability properties*, IEEE Transactions on Automatic Control **47**, pp.410-421, (2002).

[8] E.D. Sontag, *On the input-to-state stability property*, European Journal of Control, **1**, pp.24-36, (1995).

[9] A. Pavlov, A. Pogromsky, N. van de Wouw and H. Nijmeijer, *Convergent dynamics: a tribute to Boris Pavlovich Demidovich*, Systems and Control Letters, **52**, pp.257-261, (2004).

[10] B.A. Francis and W.M. Wonham, *The internal model principle of control theory*, Automatica **12**, pp.457-465, (1976).

[11] A. Isidori and C.I. Byrnes, *Output regulation of nonlinear systems*, IEEE Transactions on Automatic Control, **48**, pp. 1712-1723, (1990).

[12] A. Serrani and A. Isidori, *Global robust output regulation for a class of nonlinear systems*, Systems and Control Letters, **39**, pp. 133-139, (2000).

[13] A. Pavlov, N. van de Wouw and H. Nijmeijer, *Uniform output regulation of nonlinear systems, a convergent dynamics approach*, Birkhäuser, New York, (2006).