

Clustering Techniques Applied to Multiple-Models Structures

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Abstract: In this work a method for building multiple-model structures is presented. A clustering algorithm that uses data from the system is employed to define the architecture of the multiple-model, including the size of the region covered by each model, and the number of models. A heating ventilation and air conditioning system is used as a testbed of the proposed method.

Keywords: Pattern Identification, Identification, Pattern Recognition, Fault Identification, Process Models.

1. INTRODUCTION

The multiple-model (MM) approach allows to conveniently represent the dynamic behaviour of a nonlinear system at different operating regimes. The use of multiple models is not a new concept, it has been used since the late 1960s. In many cases where identifying a single nonlinear model proves to be difficult, the multiple-model approach has been applied successfully Maybeck and Stevens (1991). In many current industrial control systems, the regimes of operation are wide, the nonlinearities are strong and there are a large number of interactions between control loops. If the system is highly nonlinear even the most advanced nonlinear identification techniques have problems to find a good model for the range of inputs and outputs of the process.

Contrary to global models, which represent (with validity) a certain system through all the domain of inputs and outputs (DIO), local models represent with accuracy regions of the same domain. It is possible to combine local models to complement each other and form a “model” that can describe the full domain of interest. This approach has a number of advantages: the divide and conquer technique is used to solve problems in different areas with very good results; many model-based control methodologies can be easily applied using a multiple-model approach; the different local models can be built using various techniques, giving a wide degree of freedom to build the global model; the identification strategy used in the multiple-model approach brings a useful insight on the process at hand; if local and simple models are obtained then it is possible to develop simple control algorithms for each, which may improve the overall behaviour of the system when combined in an appropriate manner. Some disadvantages or challenges could also be mentioned: it is necessary to decide how to divide the regime of operation; the most appropriate number of models or their complexity is not known a priori; it is required to find a good switching mechanism between the different models.

In this work the main objective was to design an algorithm that is able to overcome one of the most difficult tasks associated with the multiple-model approach: the selection of the operating regimes and the appropriate number of models to cover the space of input and output variables. It will be presented an algorithm that will cluster similar properties of a data set in order to identify different operating regimes. The algorithm will be responsible to find the best possible global model for the system. The application of this type of algorithm in the developing of fault tolerant control systems, it will also be focused.

The paper is divided as follows: section 2 gives a brief review of multiple-model approach; section 3 introduces clustering methodologies and concepts; in section 4 the proposed strategy is presented; section 5 presents the results obtained using data from a Heating Ventilation Air-Conditioning (HVAC) system; finally some concluding remarks are given in section 6.

2. THE MULTIPLE-MODEL APPROACH

Achieving a good global model using the multiple-model approach requires the following steps: decompose the domain of inputs and outputs into several different regions that should be able to characterize the system in a global way; identify appropriate models for each region; a scheduling method needs to be chosen to change between the different regimes and models. The different steps will be over viewed in the following sub sections.

2.1 Dimensionality and coverage

The first problem in multiple-models theory is the dimensionality of the DIO and the model set that will represent it. In the multiple-model approach the objective is to divide the input and output space in small and less complex spaces. Different representations could be achieved in terms of the number of local models, the size and shape of the input-output space partitions. Because, it is the first task and will reflect the quality of the final

model, it is important to apply a correct decomposition strategy. Several options are available Johansen and Roderick (1997): decomposition into physical components; decomposition based on phenomena; decomposition based on mathematical series expansion; decomposition into goals; decomposition into operating regimes. Each of these ways of domain decomposition is not necessarily to be used on its own. Often, several of these techniques are used together in the decomposition process. In the decomposition process, finding the correct number of models in the DIO is a fundamental task. There is a trade-off between a small number of complex models where each model may represent a large region of the DIO, and large number of low complexity models that usually describe small regions. Choosing the number of models involves a trade-off between accuracy and complexity. The number of models definition and their coverage has often been done by trial and error, which may prove to be difficult. To address these difficulties a few methods have been proposed, such as the on-line adaptation of the linear models used Narendra and Xiang (2000), or choosing on-line the correct number of models from a large model set Wang et al. (2003). Nevertheless, these approaches did not solve the main problem: how to find the correct number of models and how to distribute them in the domain. In this paper this problem is addressed and a possible solution is presented.

2.2 Scheduling (Switching)

The scheduling or switching mechanism is used to decide what model or models should be used in different situations. The different switching mechanisms that are available restrict and define the relation between the different local models considered:

- 1 Soft Partitions - In the soft partition approach the local models have a weighing function that attributes a certain degree of activation to each model. The functions used in domain partition are smooth kernel functions. The models are found by minimizing the following cost function,

$$J(u, k) = \sum_{i=1}^N \int_u \|y(u, k) - \hat{y}_i(u, k)\|_2^2 g_i(u, y) du \quad (1)$$

where g_i is a smooth kernel function defined in the domain of inputs and outputs and whose range is in the interval $[0, 1]$, u is the input to the system, y and \hat{y} are the process output and the local models output, respectively. If, for all operating points $\sum_{i=1}^N g_i > 0$ and \hat{y}_i is continuous, then,

$$\hat{y}(u, k) = \sum_{i=1}^N \hat{y}_i(u, k) \tau_i(u, y) \quad (2)$$

minimizes (1) Johansen and Roderick (1997). Where τ is given by the following equation:

$$\tau_i(u, y) = \frac{g_i(u, y)}{\sum_{i=1}^N g_i(u, y)} \quad (3)$$

which is a normalization of g_i ($\sum_{i=1}^N \tau_i(u, y) = 1$).

- 2 Optimization Techniques - In some cases the correct combination of local models of the system can be found using an optimization technique. For more details see Silva et al. (2006).
- 3 Hard and Discrete Partitions - In hard partitions the switching between the local models is done in a deterministic way. At each sample time one local model is

chosen as a deterministic function of the operating point, see Johansen and Roderick (1997).

- 4 Probabilistic Approach - In this approach each model has a probability function associated with it, which indicates how appropriate the model is given the current operating conditions Y.M.Zhang and J.Jiang (2001).

2.3 Identification and Modeling

The identification and modeling of the local models is a important task, in the development of multiple-models structures. Nevertheless, due to space constraints and because this a well known subject it was left out of this overview Ljung (1999).

3. CLUSTERING

Clustering methods determine which elements in a data set are similar. These methods work by grouping data points together according to an algorithm that attempts to find centroids, around which similar data points gravitate. Clustering involves dividing a data set into mutually exclusive subgroups, without relying on predefined classes. Different clustering algorithms have been emerging through the years. In this work, two clustering methods were employed: an agglomerative technique, and a density estimation technique, whom are briefly introduced below.

3.1 Agglomerative Clustering

Agglomerative clustering is a hierarchical method that does not need a pre-defined number of partitions to start the clustering process.

The method works as follows. At the beginning every data point is considered as a cluster. Thus, when the method starts, the number of clusters is equal to the number of data points. In each step the dissimilarity between the different clusters is calculated. The objects that have a smallest dissimilarity are joined together, creating a new cluster. This process continues until all data points are grouped in a single cluster, or until a stopping criteria is reached.

The agglomerative method involves two key questions: define the agglomeration of clusters and define the stopping criterion. The first question is answered using what is usually called an dissimilarity measure (a similarity measure can also be used). The dissimilarity between different clusters can be calculated using a metric, and in this cases usually the term dissimilarity is replaced by distance. Several distance measures are available to calculate the dissimilarity between different clusters Webb (2004). After distance calculation is necessary to aggregate the different clusters and this is done using three common methods: group average; nearest neighbour; furthest neighbour. The stopping criteria can be calculated using two techniques. The interquartile range outlier detection, which is a outlier detection mechanism, based on the following equation:

$$DF \geq 6 * (Q3 - Q1) \quad (4)$$

where $Q1$ represents the first quartile, $Q3$ represents the third quartile, and DF represents the square of dissimilarity between the clusters in a certain step of the algorithm Almeida et al. (2007). Equation (4) will define in which step the agglomerative clustering will stop. The smoothing parameter is an innovative cluster separation limiter that is usually available in different

clustering techniques. It makes sense to use the smoothing parameter because when this parameter is well estimated it will reflect the density separation of the clusters. Thus, it will translate the distance separation between the different clusters. Several estimation methods can be used to determine this parameter, Webb (2004).

3.2 Density Estimation Method

Clustering techniques using density estimation with kernel functions are based on the following density function:

$$\bar{p}(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x-x_i}{h}\right) \quad (5)$$

where $\{x \in \mathbb{R}^N : x_1, x_2, \dots, x_N\}$ is a vector of N observations, K is a kernel function and h is called the smoothing parameter which can be estimated a priori, see Webb (2004).

The density function is adapted to the structure of the data based on the smoothing parameter and on the type of kernel function used. The smoothing parameter will define the intercluster distance. If h takes small values then this means that the distance between the clusters will be small because the kernel function will have a higher number of peaks. After the kernel function has been defined it is necessary to define the clusters space. One way of doing this is using the peaks of the kernel function as the cluster centers. Thus, in order to find the clusters associated with the data it is necessary to find the ‘‘valleys’’ and the ‘‘peaks’’ of the density function. One of the most used methods in finding the clusters with this approach is what is called the mean shift method (there are different methods to find the local minima and maxima of the kernel function, see for instance Yip et al. (2006)). The mean shift algorithm was first proposed by Fukunaga and Hostetler (1975), but several similar approaches have been presented in subsequent years, Comaniciu and Meer (2002); Cheng (1995); Carreira-Perpinan (2006).

Steepest descent methods may be used to locate the minima and the maxima of (5). Picking a data point from the observations vector, such a point will be successively updated using the gradient of the density function:

$$x_j^{t+1} = x_j^t + \epsilon \bar{\nabla} p(x), \quad (6)$$

where $\bar{\nabla} p(x)$ represent the gradient of the estimated kernel function and ϵ is a small positive value, iterating towards the minimum of the function. Several authors shown that the mean shift operator is an estimate of the density gradient. The mean shift can be calculated as follows Fukunaga and Hostetler (1975)Cheng (1995),

$$M(x) = m(x) - x \quad (7)$$

where $m(x)$ is the sample mean with kernel K of a data set $\{x_1, x_2, \dots, x_n\}$ at a point x , and is calculated by:

$$m(x) = \frac{\sum_{i=1}^n K(x_i - x)x_i}{\sum_{i=1}^n K(x_i - x)} \quad (8)$$

Equation (7) is used as a estimate of the density function gradient, and using (6) it is possible to find the clusters centers. For different kernel functions the mean shift will have different formulations Cheng (1995); Carreira-Perpinan (2006).

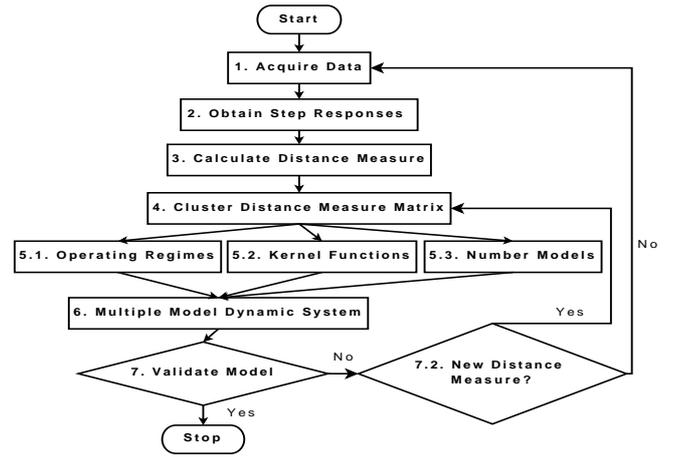


Fig. 1. Flowchart of the proposed strategy.

4. PROPOSED METHOD FOR DIMENSIONING AND COVERAGE

In this section, a clustering method is described to divide the domain of inputs and outputs of a dynamic system using measured data. The proposed method defines a simple approach to calculate a multiple-model structure that can coverage a certain interest domain in a control system. Three key stages: obtain step responses; find the cluster distance matrix; obtain kernel functions. The three stages will group the several steps necessary to obtain good results. In Fig. 1 is shown a simple flowchart where these stages are well represented. Following closely Fig. 1, the method will be explained in detail: **Step 1** The first step is to acquire data from the system. The data should be adequate to the work at hand, reflecting the different operating regimes. In this approach only step responses are considered in order to detect the different operating regimes. Thus, the identification data should be composed of several step responses in other to excite different regimes and to excite different frequencies.

Step 2 After the data has been acquired, it is necessary to obtain information about the different dynamics of the system. In this work two different strategies to obtain information from the step responses have been tested: using a symbolic representation of the data set to discretize and analyze the acquired data; a step response reference. The symbolic representation used was the *symbolic aggregate approximation* (SAX). With the SAX method the data set is reduced from n dimensions to w dimensions, where w represents the time window to be considered. Each time window is replaced by the calculated mean in that time interval. This technique is called Piecewise Aggregate Approximation (PAA). Afterwards, the time series is broken in equiprobability spaces and a letter is attributed to each one of the spaces (creating strings representing the data set). After the application of the SAX method the data will be described by a number of separated and normalized step responses that are characterized by strings. The SAX method is described in detail in Lin and Chiu (2003). After this step the dynamic responses are normalized, having the same length and can be characterized by strings. The other strategy to obtain dynamic information is the step response reference. A step response will be used as a reference and like a pattern will be compared to each one of the others step responses. The two strategies can be used together in order to obtain more features in further steps.

Step 3 When the step responses are isolated, different distance measures can be applied in order to define and characterize them. The distance between the different step responses will be the feature available to the clustering algorithms. Two different distance measures were used in this work: Euclidean distance dissimilarity measure; the Hamming dissimilarity measure. These two measures are used accordingly with the type of analysis that is conducted in the step responses. The Euclidean distance is used when a step reference is available and the Hamming dissimilarity is used after the SAX method. In this sense, when possible, the proposed strategy will create two different features that will help the clustering algorithm. Also, the fact that two features are available will make the clustering algorithm more robust to problems in the acquisition of the data (noise and outliers).

Step 4 Using the distance matrix, the clustering algorithm will find the number of models that should be used to represent the system, the location and range of the linear models in order to cover the input-output domain. The clustering algorithms will find the step responses that are similar, and in this sense, the operating regimes. In this proposed method two clustering algorithms were tested: agglomerative approach; density clustering using the mean-shift operator. The clustering methods will give as an output the clusters identified in the distance matrix. Each one of the clusters identified will be responsible for the representation of an operating regime. Since, different clustering techniques are available and can be used, the most important aspect is the ability of the clustering technique to define the optimum number of clusters without any initial restriction, making the method completely blind to the initial guess of the number of operating regimes.

Step 5 As a result the clustering algorithm will output the number of models necessary to describe the different operating regimes (number of clusters) and the kernel functions that partition the input-output domain. When the clusters are identified the center of each one is calculated and will represent a specific operating regime. Using this information it is possible to build switching functions to automatically develop a switching mechanism, based on soft partitions, that will represent the global system using a multiple-model approach.

Step 6 The information given by the clustering algorithms is used to acquire data of the system, but now concentrating in the operating regimes decided by the clustering algorithms. After step 5 sufficient information to search for the correct linear models in the domain has been achieved. Knowing where the most important linear dynamics are concentrated and knowing how many models should be used and where they should be located is important to identify the multiple-model structure. In this step two different situations can occur: the data acquired is sufficient to try to identify the different local models; new data must be acquired from the process (depends on the quality of the data acquired for the operating regimes calculated in step 5). When the linear models are built to represent each of the operating regimes together with the kernel functions obtained in step 5, a multiple-model is available to represent the dynamic behaviour of the system.

Step 7 Finally the multiple-model should be validated against new data acquired from the system. If the model is not accurate the process should start again. At this point two hypotheses must be considered: the data acquired is not the most representative and another experimental test should be conducted with

different excitation signals; the clustering algorithm did not behave as expected and the operating regimes are not the best ones. In this case the clustering algorithms should be executed again changing the input variable (for instance the smoothing parameter or the outliers rejection equation).

With the proposed method it is possible to build a multiple-model structure automating some of the most difficult decisions in the multiple-model approach: dimensionality and coverage.

5. RESULTS

In this section some results obtained with the proposed method will be presented. The proposed approach is applied to a Heating Ventilation and Air Condition (HVAC) system.

5.1 The HVAC system

A real HVAC system with a Air Handling Unit (AHU) and of Variable Air Volume (VAV) used in the Engineering Building of the University of Reading, is going to be used as a testbed of the proposed technique. This HVAC system is used to regulate air conditions in three different rooms. The HVAC system is composed of several components: cooling coil, heater bank, fan, terminal units, etc.

Apart from the mechanical components, this system has a very exhaustive instrumentation apparatus to collect data and control the actuators. A Local Operating Network (LON) communication structure is installed to acquire data from sensors and to send data to actuators. All the signals are received in a central server. This server is implemented using *Microsoft Windows® Server 2000*. The information collected from the system is available through a Dynamic Data Exchange (DDE) server. Using the DDE protocol all the signals are acquired in *Matlab®* and *Simulink®* is used to execute the control algorithms. The sampling time used in all tests performed was 1 second.

5.2 Terminal Units

The terminal units are positioned in the three rooms where the air is conditioned. These terminal units regulate the air flow that is inserted in each of the rooms. Each of the terminal units has a internal Proportional-Integral (PI) controller that is used to position its blades. The angle of the blades determines the flow in the terminal unit and consequently the flow entering the rooms. Each terminal unit has a flow sensor and a position sensor (which measure the air flow and the blade angle, respectively) that can be accessed by the LON network. It is possible to change the position of the blade by sending a reference signal through the network.

The HVAC supply fan receives a signal of $[0, 10]V$, which provides a range of air flow available in the terminal units between 0 l/s and 500 l/s. To model the terminal units the reference signal to the terminal unit blade and the signal to the HVAC fan can be used as inputs, and the flow entering the air conditioned rooms can be used as outputs. In this particular case the multiple-model approach will try to identify different local linear models in respect to fan regime. Changing the fan input signal will change the dynamic behaviour of the terminal unit. It is necessary to find the number of local models needed to model the global process and is necessary to know their

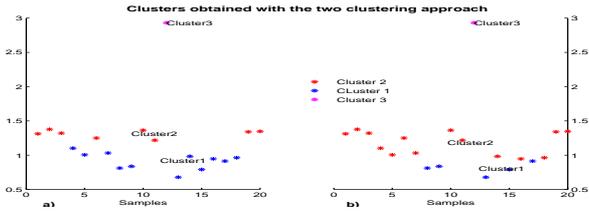


Fig. 2. Clusters obtained in the acquired data. a) Agglomerative Clustering; b) Mean-Shift Clustering.

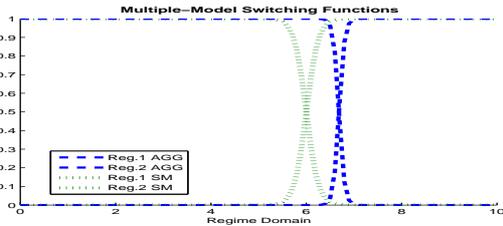


Fig. 3. Switching function obtained with the different clustering techniques: AGG - Agglomerative Method; SM - Mean-Shift Method.

location. The proposed method for dimensioning and coverage has been applied to this process.

The operating regimes of the terminal unit are defined by the changes in the operating regime of the supply fan. The two clustering algorithms described earlier were tested in this approach. Both algorithms were executed using the smoothing parameter as the stopping criteria. For both approaches the smoothing parameter used was 0.18 (it was obtained using techniques described by Webb (2004)). As can be seen in Fig. 2 a) and b) the two clustering algorithms provided very similar results and both identify three clusters. Cluster number three is clearly an error in the measured data because is outside the mean values presented on the clustering distance matrix. The two clustering techniques yielded similar results in terms of the switching functions obtained, as is depicted in Fig. 3.

The proposed method for dimensioning and coverage indicated that the model for the terminal units should be divided in two linear models, that should be centered at 7.5 and 6.5 values of the operating regime of the supply fan. With this information, identification experiments were performed focusing on these regimes and two linear models were obtained. The results were obtained using the agglomerative method and the soft partitions approach. The results obtained with the validation data can be seen in Fig. 4. The mean square error obtained in this test was 60.0. It is important to note that before using the proposed strategy to built the multiple-model structure, a multiple-model representing the terminal units was built using empirical information (three linear models) and empirical kernel functions. Using the later model in the validation data the mean square error obtained was 63.3 (see Silva et al. (2006)).

6. HEATER BANK

6.1 Heater Modeling and Identification

The electric heater bank is used to heat the air supply temperature. The amount of heating power produced in the electric bank can be regulated by an analog control signal [0,10]V. The heater is a nonlinear component and the nonlinearity is

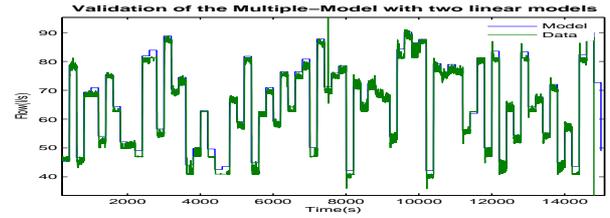


Fig. 4. Terminal unit multiple-model validation data

extremely evident with the variation of flow that is crossing the heater. The stationary regime and the transient regime will suffer variations when the airflow crossing the heater changes. Using an approach similar to the terminal units case, data is acquired from the process and the algorithm to automatically obtain the ideal number of models for the operating regime is executed. Using some of the concepts presented earlier the smoothing parameter used was 0.072, and was calculated based on the Biased Cross-Validation (BCV) method Webb (2004). The number of models obtained in the clustering algorithm was 5 models. The clustering algorithm used was the mean-shift algorithm using a flat kernel function. The new linear models were focused on the regimes obtained in the clustering algorithm. The five models were centered in the following zones: $AF_1 = 80l/s$; $AF_2 = 170l/s$; $AF_3 = 250l/s$; $AF_4 = 330l/s$; $AF_5 = 400l/s$. In order to validate the multiple-model structure developed, another multiple-model structure was designed based on the empirical knowledge of the system, with 3 local linear models (in this case the regimes and the partition functions were obtained by empirical knowledge and trial and error). In Figure 5 it is presented the membership functions that represent the partition of the domain space for the 3 models (Figure 5-a) and for the 5 five models (Figure 5-b), obtained with the proposed method). A simulation was made using the two multiple-model structures, see Figure 6. The mean square error obtained for the simulation of the different structures is presented in Table 1.

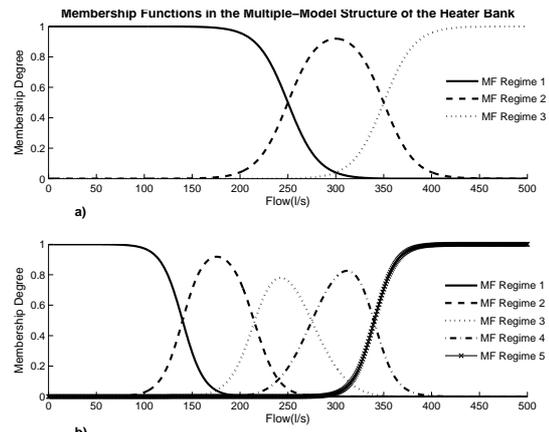


Fig. 5. Membership functions used on the multiple-model structure with: a) 3 models; b) 5 models.

| Multiple-Model Structure | MSE |
|--------------------------|--------|
| 3 Models | 1.9000 |
| 5 Models | 1.0273 |

Table 1. Mean Square error for the multiple models structures of the heater bank.

Using all the information collected, the heater bank can be accurately represented using a multiple-model structure with

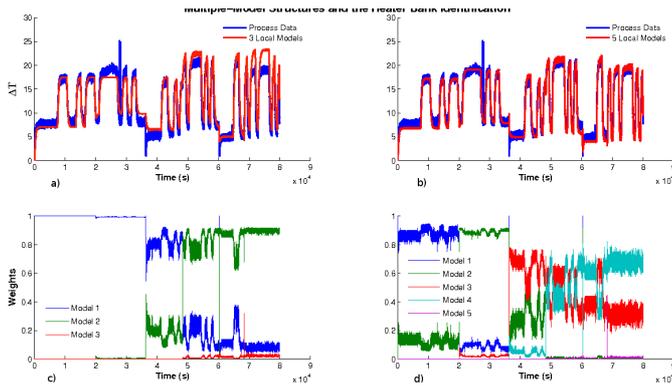


Fig. 6. Results obtained with the two multiple-model structures, and regimes combinations calculated during the simulations. a) Simulation results for a multiple-model structure with three models. b) Simulation results for a multiple-model structure with five models. c) Switching in the three models structure. d) Switching in the five models structure.

five operating regimes and a local linear model in each one of the regimes. The proposed method could be improved and a better multiple-model structure for this process could be obtained.

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7. CONCLUSION

In this work a simple but effective strategy was proposed to automatically partition the domain space of the input-output variables. The method is based on clustering techniques and soft partitioning by means of kernel functions. The method was tested with a real HVAC system and the results obtained are encouraging.

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