Directional Distribution-based Mutation for Evolutionary Algorithms

Andrzej Obuchowicz * Przemysław Prętki **

* Institute of Control and Computation Engineering, University of Zielona Góra, Zielona Góra, Poland (e-mail: a.obuchowicz@issi.uz.zgora.pl)
** Institute of Control and Computation Engineering, University of Zielona Góra, Zielona Góra, Poland (e-mail: p.pretki@issi.uz.zgora.pl)

Abstract: In this paper, a concept of directional mutations for phenotypic evolutionary algorithms is presented. The proposed approach allows, in a very convenient way, to adapt the probability measure underlying the mutation operator during evolutionary process. Moreover $\alpha$-stable distributions are used to generate the mutation radius. A correlation analysis between control parameters -- stability index $\alpha$, slope parameter $\sigma$ and concentration parameter $\kappa$ -- is done using simulating experiments.

Keywords: Directional mutation, evolutionary algorithms, $\alpha$-stable distributions, global optimization.

1. INTRODUCTION

Mutation operators based on the isotropic distributions are the most popular in Evolutionary Algorithms (EA), which use the floating-point representation for individuals, e.g., Evolution Programming (EP) (Fogel et al. (1991)) and Evolutionary Strategies (ES) (Beyer and Schwefel (2002)). Undoubtedly, probabilistic models of the spherical symmetry possess both advantages and disadvantages. One of important advantages is small number of parameters which defines them unambiguously. In the case of static algorithms, this fact allows significant simplification of an optimization process. In the case of adaptive algorithms, a heuristics fitting exploration distribution to the environment can be determined in simpler way. The small number of parameters possesses also negative consequences, which contribute to the searching effectiveness decreasing. The efficacy of stochastic optimization techniques significantly depends on the high ability of searching model adaptation to a considered problem. The small number of free parameters restricts the adaptation leeway to a narrow class of models. The spherical exploration model can respect the correlation between an objective function and Euclidean distance form the base point, only. If there exists strong correlations between decision variables of the problem optimized, the isotropic exploration distribution application usually prolongs a time of searching.

Problems described above can be eliminated by substituting isotropic exploration models by non-isotropic ones. There are many proposals of EAs with the mutation based on the elliptical normal distribution (Beyer and Schwefel (2002); Beyer and Arnold (2003); Hansen and Ostermeier (2001)). These algorithms are usually enriched by adaptation procedures, which the imperative goal is the model fitting to currently exploited area of the optimization environment. However, the mutation operator based on the elliptical normal distribution $\mathcal{N}(\mu, \Sigma)$ allows to model the relationship between the decision variables, it has a disadvantage. Both directions, parallel to the vector of expected improvement and its opposite direction, possess the same probability of selection. This fact manifests in slow convergence of the EA. This problem will be described in details in one of subsequent subsections.

2. ISOTROPIC MUTATIONS

Let us quote the definition (Fang et al. (1990)):

Definition 1. Let $X^* = (X_1, ..., X_n)^T$ be an $n$-dimensional random vector, $O(n)$ be a class of orthogonal matrices of $n \times n$, and $u^{(n)}$ be a random vector of uniform distribution over the $n$-dimensional unique sphere. The random vector $X^*$ is of spherical symmetry distribution if and only if one of the following conditions is met:

1) $X^* \stackrel{d}{=} O X^*$ for each matrix $O \in O(n)$,
2) the characteristic function of $X^*$ has the form $\phi(\theta^T \theta)$, where $\phi(\cdot)$ is a some scalar function called the characteristic generator,
3) $X^*$ can be decomposed to the form $X^* \stackrel{d}{=} r u^{(n)}$, where $r \geq 0$ is a some random value independent on $u^{(n)}$,
4) $a^T X^* \stackrel{d}{=} ||a|| x_1$ for each $a \in \mathbb{R}^n$.

2.1 Curse of dimensionality

Let us consider the relation 3 in the above definition, which describes the relation $1 \rightarrow 1$ between $r$ and $X^*$, i.e., the set of all isotropic distributions and the set of all positive distributions is of the same cardinality (Fang et al. (1990)). It is worth to notice, that the vector $u^{(n)}$ has the largest possible random entropy over all distributions defined on the unique sphere. This fact is the most often quoted reason of the normal distribution application to stochastic optimization algorithms in $\mathbb{R}^n$. 
However, this feature seems to be the most desirable, but, in practice, it has a negative influence on the effectiveness of the optimization algorithms. If one assumes that only the sparse subset of elements of a space on which the distribution is defined possesses desire features, the largest entropy simultaneously describes the smallest chance of selection of any element from this subset. In the case of stochastic optimization in $\mathbb{R}^n$, especially in the case of very large $n$, if the uniform distribution over the $n$-dimensional unique sphere is applied to selection of a mutation direction then the probability that the chosen direction will be parallel to the direction of an objective function improvement can become very low.

Above intuitive consideration can be supported by the following theorem:

**Theorem 2.** Let $X^n$ be a random vector of a spherical symmetry. We assume that an alternative solution is generated by the formulae: $x_{t+1} = x_t + X^n$. Let $\mu_t$ be the most favorable direction of mutation of the base point $x_t$. Thus, the probability that $x_{t+1}$ will be perpendicular to $\mu_t$ converge to 1 when $n$ converge to infinity.

The proof can be found in Prętki (2008).

The theorem 2 allows to explain and, moreover, quantitative approximate the decrease of evolutionary algorithms efficiency in high-dimensional problems. Figure 1 illustrates the above theorem.

![PDF](image.png)

**Fig. 1.** The density function of the random variable $t = x^T \mu$ for the isotropic random vector $x$ and the most favorable direction of mutation $\mu$.

### 2.2 Distribution symmetry problem

The elliptical normal model is the most often applied non-isotropic exploration distribution in EAs (Fang et al. (1990)). In the case of multidimensional normal distribution $N(0, \Sigma)$ the term ”elliptical” corresponds to the fact, that $(n - 1)$-dimensional isosurfaces of $n$-dimensional density function of $N(0, \Sigma)$ are hyper-ellipsoids with equatorial radii parallel to the eigenvectors of the covariance matrix $\Sigma$. One of the most important problem connected with these distributions is designing an effective procedure of the covariance matrix selection during the evolutionary process. Very rich literature concerns this problem (e.g. Hansen and Ostermeier (2001); Schwefel (1981)). However, researches propose different heuristics and techniques of information collection and processing, but the aim of all these procedures is fitting of the exploration distribution in such a way that the area of better fitness quality will be sampled more intensively. The multidimensional normal distribution approach is connected with some risk, which is commented very seldom by researches testing this type of solutions. Processes of the $\Sigma$ adaptation, better or worse, leads to establishing the major radius parallel to direction of the best improvement of the fitness function. Such ideal situation for spherical function $\phi(x) = x^T x$ and the base point $x_b = [2, 2]^T$ is presented in figure 2. It can be seen, that the symmetry of the distribution causes ration of the same probability mass for the best as well as the worst distance of the fitness improvement. This means, in general, that more then the half of random disturbances of the base point will fail. This fact is one of causes of low efficiency of stochastic algorithms in environments of very high dimensionality.

### 3. DIRECTIONAL MUTATION

#### 3.1 Directional distribution

The general idea of directional exploration distributions is based on a stochastic decomposition, as it has been done in the case of isotropic distributions (Def. 1). The crucial difference is that the isotropic base distribution $u^{(n)}$ is replaced by the diversified distribution on the unit sphere surface $d^{(n)}$ (Prętki and Obuchowicz (2006))

$$Z \sim r\,d^{(n)}$$

(1)

Let $X$ be a random vector of the rotational symmetry. Rotational symmetry is represented by distribution invariance in relation to rotations round a given direction. $X$ can be stochastically decomposed in the following way (Mardia and Jupp (2000)):

$$X = t\theta + \sqrt{1 - t^2}\xi,$$

(2)

where $t$ is a random variable which is invariant in relation to rotations round the direction $\theta$, $\xi$ is a random variable of the uniform distribution on the surface of the unit sphere $\partial S_{n-2}(0)$. Moreover, $\xi$ and $t$ are mutually independent.

The decomposition (2) will be used to construction of the distribution of rotational symmetry with any marginal.
distribution. In order to do it, let \( \theta = [0, 0, \ldots, 1]^T \in \mathbb{R}^n \) in (2), and \( QX \) be an orthogonal transformation that \( Q\theta = \mu \). It is easy to prove that the obtained in this way random vector \( X \) has a marginal distribution \( X^T \theta \overset{d}{=} t \). One can control the distribution of the probability mass focused round a expected direction \( \mu \) by a suitable selection of the distribution of \( t \) (Prętki and Obuchowicz (2006)). In our approach we recommend the marginal distribution in the form \( t = 2X - 1 \), where \( X \) is a random variable of the Beta distribution \( \beta(a, b) \) (Shao (1999)). Thus, the probability density of \( t \) has the form (Prętki and Obuchowicz (2006))

\[
f(t|a, b) = \frac{2^{1-a-b}}{\beta(a, b)} (1 - t)^{b-1}(1 + t)^{a-1},
\]

where \( a = \frac{n-1}{2} \) and \( b = \frac{\kappa(n-1)}{2} \) be parameters of the Beta distribution, \( n \) is a space dimension and \( \kappa \) is a so called, concentration parameter. The expectation value of the random variable \( T = \theta^T X \) and its variance have the form:

\[
E[T] = \frac{1 - \kappa}{1 + \kappa}, \quad \text{Var}(T) = \frac{8\kappa}{n(1 + k)^2(1 - \kappa^2)}.
\]

The directional distribution simulation process is summarized in the Tab.1.

Table 1. Algorithm to simulating directional distribution \( M(\mu, \kappa) \)

<table>
<thead>
<tr>
<th><strong>Input data</strong></th>
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<tbody>
<tr>
<td>( \mu \in \mathbb{R}^n ) – mean direction</td>
<td></td>
</tr>
<tr>
<td>( \kappa \in (0, 1) ) – concentration parameter</td>
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</table>

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<tr>
<th><strong>Output data</strong></th>
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<tr>
<td>( Y ) – pseudo-random vector of ( M(\mu, \kappa) ) distribution</td>
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<tr>
<th><strong>Algorithm</strong></th>
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<tbody>
<tr>
<td>( t = 2\beta(\frac{n-1}{2}, \frac{\kappa(n-1)}{2}) - 1 ), where ( \beta(a, b) ) gives random number from Beta distribution</td>
<td></td>
</tr>
<tr>
<td>( X \leftarrow \mathcal{N}(0, I_{n-1}) )</td>
<td></td>
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<tr>
<td>( Z \leftarrow X/|X|_2 )</td>
<td></td>
</tr>
<tr>
<td>( Y \leftarrow [\sqrt{T-\frac{2}{T}}^T, q]^T )</td>
<td></td>
</tr>
<tr>
<td>( Y \leftarrow [I_n - 2\nu v^T] Y ) where ( v = \frac{[0, 0, \ldots, 1]^T - \mu}{| [0, 0, \ldots, 1]^T - \mu |_2} )</td>
<td></td>
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</tbody>
</table>

Figure 3 presents realizations of the proposed directional distributions (Algorithm 1) for different values of \( \kappa \)

### 3.2 Mean direction selection

The effectiveness of evolutionary algorithms with mutation based on a class \( M(\mu, \kappa) \) will depend at least on two factors: the correctness of establishing the mean direction of mutation \( \mu \), and the value of the concentration parameter \( \kappa \), which controls the dispersion around the mean direction. In fact, concentration parameter \( \kappa \) allows to obtain on one side an isotropic distribution on the sphere \( (\kappa = 1) \), and on the other side, a degenerate distribution at the mean direction \( (\kappa = 0) \). The idea of forcing mutation direction boils down to utilizing a traditional way of creating a new individual. Since directional distributions, are parameterized by a pair \( (\mu, \kappa) \), then one must determine the strategy of adjusting their values. In this paper, the

\[
\text{Fig. 3. Realizations of } 10^5 \text{ pseudo-random vectors of } M(\mu, \kappa) \text{ for } \mu = [-1, 0, 0]^T \text{ and } \begin{cases} (a) \rightarrow \kappa = 0.001, \\ (b) \rightarrow \kappa = 0.01, \\ (c) \rightarrow \kappa = 0.1, \\ (d) \rightarrow \kappa = 1 \end{cases}
\]

attention is restricted to the parameter \( \mu \) only. In the literature, several techniques doing this can be found:

- **Heuristic No. 1** – the most promising direction is given by the formula:
\[ \mu^t = -\frac{z}{\|z\|_2}, \quad \text{where} \quad z = [P_t^T P_t]^{-1} P_t^T \Phi_t, \]

where \( P_t \in \mathbb{R}^{n \times n} \) and \( \Phi_t \in \mathbb{R}^n \) stands for a matrix of phenotypes and a vector of individual fitness respectively, e.g.:

\[ P_t = [x_1^t, x_2^t, \ldots, x_n^t]^T \]
\[ \Phi_t = [\phi(x_1^t), \phi(x_2^t), \ldots, \phi(x_n^t)]^T \]

An obvious drawback of the method lies in the fact that the inversion of the matrix \( P_t^T P_t \) requires at least as many individuals as the dimension of the search space.

- **Heuristic No. 2** – the method, firstly proposed by Salomon (1998), for the class of algorithms known as Evolutionary Gradient Search:

\[ \mu^t = \frac{z}{\|z\|_2}, \quad (5) \]

where

\[ z = \sum_{k=1}^{n} \frac{\phi(x_k^{t-1}) - \phi(x_k^t)}{\phi(x_k^{t-1}) - \phi(x_k^t)} \|x_k^{t-1} - x_k^t\|_2 \]

- **Heuristic No. 3** – the approach proposed by Obuchowicz (2003) for the class of evolutionary algorithms known as Evolutionary Search with Soft Selection and Forced Direction of Mutation (ESSS-FDM):

\[ \mu^t = \frac{\langle x^t \rangle - \langle x^{t-1} \rangle}{\|\langle x^t \rangle - \langle x^{t-1} \rangle\|}, \quad \text{where} \quad \langle x^t \rangle = \frac{1}{\eta} \sum_{k=1}^{n} x_k^t \]

### 3.3 Distribution of the mutation radius

The selection of mutation radius \( r > 0 \) in (1) is another important problem which should be solved. Wide discussion about this problem can be found in Obuchowicz (2003). In our approach the Levy-stable distributions are applied (Obuchowicz and Prętki (2004); Samorodnitsky and Taqqu (1994)). The ch.f. (characteristic function) of the \( \alpha \)-stable distribution is parameterized by a quadruple \((\alpha, \beta, \sigma, \mu)\) (Samorodnitsky and Taqqu (1994)), where \( \alpha \) (0 < \( \alpha \) ≤ 2) is a stability index (tail index, tail exponent or characteristic exponent), \( \beta \) (−1 ≤ \( \beta \) ≤ 1) is a skewness parameter, \( \sigma \) (\( \sigma \) > 0) is a scale parameter and \( \mu \) is a location parameter. The lack of closed-form formulas for probability density functions (pdfs) for all but three LSDs (Gaussian, Cauchy and Levy distributions) constitutes a major drawback while using \( \alpha \)-stable distributions in practice. Fortunately, there exist algorithmic formulas for simulating \( \alpha \)-stable variables as well as computer programs to compute \( \alpha \)-stable densities, distribution functions and quantiles (Nolan (2003)).

If \( \beta = 0 \), then non-skewed \( \alpha \)-stable distributions, called the Symmetric \( \alpha \)-Stable (\( SaS \)) distribution, is obtained. Thus, \( Z \sim S_{\alpha}(0, \sigma, \mu) = SaS(\sigma, \mu) \) (symmetric \( \alpha \)-stable) can be expressed by

\[ Z = \mu + \sigma X, \quad (6) \]

where \( X \sim S_{\alpha}(1, 0) = SaS \) has standardized symmetric \( \alpha \)-stable distribution. The ch.f. of \( X \) is given by

\[ \phi(k) = \exp(-|k|^{\alpha}). \quad (7) \]

For \( \alpha = 1 \), it is a ch.f. of the Cauchy distribution \( C(0, 1) \), and for \( \alpha = 2 \), it becomes the ch.f. of the normal distribution \( N(0, 1) \).

In order to guarantee positive values of the mutation radius, it is chosen as a realization of the random value of the distribution \( \chi_{\alpha, \sigma} \equiv |SaS(\sigma)| \)

### 4. SIMULATION EXPERIMENTS

#### 4.1 Evolutionary Search With Soft Selection

Evolutionary algorithms used in simulation experiments in this work are based on the ESSS algorithm (Evolutionary Search with Soft Selection), which is based on a probably the simplest selection-mutation model of the Darwinian’s evolution (Galar (1989); Karcz-Dulęba (2004)). To stress that original algorithm is modified by applying directional mutation and the Levy-stable distribution the abbreviation ESSS_\( \alpha \) - DM is used. The evolution is a motion of individuals in the phenotype space, called also the adaptation landscape. This motion is caused by the selection and mutation process. Selection leads to the concentration of individuals around the best ones, but mutation introduces the diversity of phens and disperses the population in the landscape. The above-described assumptions can be

| Table 2. Outline of the ESSS_\( \alpha \) - DM algorithm |

| Input data |
| \( \eta \) – population size; |
| \( t_{\text{max}} \) – maximum number of iterations (epochs); |
| \( \sigma, \alpha, \kappa \) – parameters of mutation; |
| scale, stable index and concentration; |
| \( \phi: \mathbb{R}^n \rightarrow \mathbb{R} \) – fitness function; |
| \( x_{k0} \) – initial point. |

**Algorithm**

1. Initialize

\[ P(0) = (x_1^0, x_2^0, \ldots, x_n^0), \quad x_k^0 = x_{k0} + Z, \]

where \( Z \sim N(0, \gamma I_n) \), \( k = 1, 2, \ldots, \eta \)

2. Repeat

   (a) Estimation

   \[ \Phi(P(t)) = (q_1^t, q_2^t, \ldots, q_\eta^t), \]

   where \( q_k^t = \phi(x_k^t) \), \( k = 1, 2, \ldots, \eta \).

   (b) Tournament selection

   \[ P(t) \rightarrow P(t') = (x_{h_1}^t, x_{h_2}^t, \ldots, x_{h_\eta}^t), \]

   (c) Estimation of the most promising direction of mutation

   \[ \mu(t) \leftarrow H(P'(t), P'(t-1)) \]

   (d) Mutation

   \[ P'(t') \rightarrow P(t+1); \quad x_k^{t+1} = x_{hk}^t + \chi_{\alpha, \sigma} u^{(n)} \]

   \[ u^{(n)} \leftarrow M(\mu(t), \kappa), k = 1, 2, \ldots, \eta \]

   Until \( t > t_{\text{max}} \).

Formalized by the algorithm presented in Tab. 2.

#### 4.2 Benchmark optimization functions

Four 3D testing functions are chosen for simulating experiments:
• spherical model - $\phi_1$
  $$\phi_1(x) = \sum_{i=1}^{30} x_i^2$$  \hfill (8)

• ellipsoidal function - $\phi_2$
  $$\phi_2(x) = \sum_{i=1}^{30} \left( \frac{x_i^2}{100} \right)^{\frac{1}{\kappa}}$$  \hfill (9)

• Rastrigin’s function - $\phi_3$
  $$\phi_3(x) = 300 + \sum_{i=1}^{30} \left( x_i^2 - 10 \cos(2\pi x_i) \right)$$  \hfill (10)

• Ackley’s function - $\phi_4$
  $$\phi_4(x) = -20 \exp \left( -0.2 \sqrt{\frac{1}{30} \sum_{i=1}^{30} x_i^2} \right) - \exp \left( \frac{1}{30} \sum_{i=1}^{30} \cos(2\pi x_i) \right) + 20 + e$$  \hfill (11)

Two first function $\phi_1$ and $\phi_2$ are unimodal, but $\phi_2$ is ill-

conditional. Functions $\phi_3$ and $\phi_4$ are strongly non-linear and multimodal.

4.3 Control parameters

Our experiment needs to fit three control parameters $\alpha, \sigma, \kappa$ and heuristic of mean direction selection for each of benchmark optimization problems. In order to analyze the correlation between all of these parameters, a set of experiment are done with each combination of the following sets of values:

- $\alpha = \{2.0, 1.75, 1.5, 1.25, 1.0, 0.75, 0.5\}$,
- $\sigma = \{0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10\}$,
- $\kappa = \{0.01, 0.1, 0.25, 0.5, 0.75, 1\}$.

Each configuration of parameters $\{\alpha, \sigma, \kappa\}$ with one of chosen heuristics has been evaluated taking account 100 algorithm runs. Initial populations have been obtained by multiple disturbance of an initial point $x_0$ using normal distribution $N(0, I_n)$. The population size $n = 50$ was chosen taking into account the inverse operation of the heuristic 1.

4.4 Results

The best configurations of parameters for each heuristics and objective functions are presented in Table 3. Tables 4 and 5 shows the correlation ratio for explanatory variables $\alpha, \sigma$ and $\kappa, \sigma, \kappa$, respectively, and three responses variables $Me[H_1], Me[H_2], Me[H_3]$.

Application of the tournament selection with a relatively big size of the tournament group is a cause of a big selection pressure - the best individualshave very large probability to be kept in population. This fact influences on the parameters values of the best their configurations (Tab.3). It can be seen, that values of $\kappa$ are not small, there are not strong pressure to choose the favored direction. In two cases, the isotropic distribution occurs to be the best. The conclusion is: the directional mutation is not favorable for evolutionary algorithms with elitist selections. It is worth to noticing that, unlike in the case of proportional selection (Obuchowicz (2007)), the optimal exploration distributions are based on the stability indices $\alpha < 2$.

5. CONCLUSIONS

In this paper, the general concept for the adaptation of a probabilistic measure in a mutation operator of phenotypic EAs is considered. The proposed approach is based on the directional distributions which are parameterized by the mean vector and concentration parameter. Simulated
Table 5. Correlation ratio for explanatory variables $\kappa, \sigma$ and three responses variables $\text{Me}[H_1], \text{Me}[H_2], \text{Me}[H_3]$.

<table>
<thead>
<tr>
<th>Objective function $\phi_1$</th>
<th>var.</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.16</td>
<td>0.22</td>
<td>0.14</td>
<td>-0.55</td>
<td>-0.65</td>
<td>-0.58</td>
<td></td>
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<table>
<thead>
<tr>
<th>Objective function $\phi_2$</th>
<th>var.</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.93</td>
<td>0.81</td>
<td>0.69</td>
<td>0.67</td>
<td>0.63</td>
<td>0.69</td>
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<tr>
<th>Objective function $\phi_3$</th>
<th>var.</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
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<tbody>
<tr>
<td>$\kappa$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.91</td>
<td>0.12</td>
<td>-0.26</td>
<td>-0.40</td>
<td>0.01</td>
<td>-0.22</td>
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<thead>
<tr>
<th>Objective function $\phi_4$</th>
<th>var.</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
<th>H_1</th>
<th>H_2</th>
<th>H_3</th>
</tr>
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<tbody>
<tr>
<td>$\kappa$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.16</td>
<td>-0.05</td>
<td>-0.16</td>
<td>0.12</td>
<td>-0.08</td>
<td>-0.14</td>
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Experiments confirm the thesis that the proposed mutation improves the effectiveness of evolutionary algorithms in the case of the local as well as global optimization problems. It also enables constructing more sophisticated techniques which aim at improving effectiveness of evolutionary algorithms for high-dimensional problems.

REFERENCES


