Supervisory advanced control and on-line set-point optimization*

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Abstract: The subject of the paper is to discuss certain effective and novel structures for advanced process control and optimization. First the role and techniques of model-based predictive control (MPC) at supervisory control layer (advanced control layer) of the multilayer process control structure will be shortly reviewed, with the emphasis on efficiency in implementation of algorithms for nonlinear processes and on treating uncertainty in process models. Issues of cooperation between MPC supervisory algorithms and on-line steady-state set-point optimization will be next discussed, including integrated approaches. Finally, a recently developed two-purpose supervisory predictive algorithm capable to generate set-points for underlying unconstrained direct controllers with the goals to optimize economically and to handle constraints will be discussed.

Keywords: Predictive control, set-point optimization, constrained control, multivariable control, nonlinear control, linearization.

1. INTRODUCTION

The hierarchical (multilayer) approach to process automation is a standard in process industries and a well understood technique able to cope with complexity and multiple criteria of operation. The main control layers are: the regulatory (feedback) control layer, which keeps process at given operating points and can itself be divided into basic and advanced control layers and the set-point optimisation layer, which calculates these operating points (Findeisen, et al., 1980; Findeisen, 1997; Blevins, et al., 2003, Brdys and Tatjewski, 2005; Tatjewski, 2007).

In complex control systems applying the advanced control techniques the regulatory control layer consists typically of two layers: the basic (direct) dynamic control layer, usually equipped with PID controllers, and a higher, advanced control layer (called also constraint control layer, set-point control layer or MPC layer), in which Model Predictive Control (MPC) algorithms are typically implemented, see e.g., (Kassmann, et al., 2000; Blevins, et al., 2003; Qin and Badgwell, 2003; Maciejowski, 2002, Brdys and Tatjewski, 2005; Tatjewski 2007, 2008). The most important advantage of the MPC algorithms is the fact that they have the unique ability to take into account constraints imposed on process inputs (manipulated variables) and outputs (controlled variables) or state variables. Moreover, MPC technique is very efficient when multivariable control is important (processes with strong interactions) and, generally, for processes with difficult dynamics. These properties have usually crucial influence on quality, economic efficiency and safety of production (Qin and Badgwell, 2003; Blevins, et al., 2003; Camacho and Bordons, 2004; Maciejowski, 2002; Rossiter, 2003; Tatjewski, 2007, 2008). The control structure incorporating the discussed layers is presented in Fig. 1, where the set-point optimization layer has been denoted as a *local set-point optimization* connected with the actual controlled part of the plant and a plant-wide optimization /management layer has also been added.

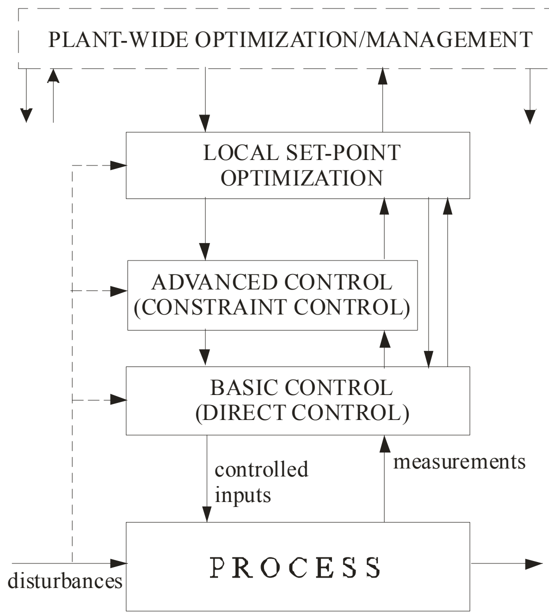

Fig. 1. The multilayer control structure

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The detailed discussion of the multilayer control hierarchies is beyond the scope of this paper, see elsewhere, e.g., (Blevins, et al., 2003; Brdys and Tatjewski, 2005; Tatjewski, 2007) and also (Skogestad, 2000, 2004; Zheng, et al., 1999) for topics such as selection of manipulated and controlled variables, controller types, etc. We shall only mention that the structure of Fig. 1 stems from a *functional decomposition*, as it is based on assigning a set of functionally different partial control objectives in a structure of verticaly dependent layers. These partial objectives are:

1. To maintain the process in a safe operation mode, *i.e*., to constrain to an acceptable level the probability of undesirable, uncontrollable process behaviour (caused by disturbances, faults, *etc.*) – *the direct control layer*,
2. To meet demands on product quality and economical usage of technological apparatuses, *i.e.*, keeping certain process input and output values on or within prescribed limits – *the advanced control layer*,
3. To maximize the current production profit – *the local optimization layer*.

Let us notice that history and significance of the advanced control layer (constraint control layer) is directly connected with the development of advanced control algorithms, in fact almost exclusively the MPC algorithms. There was no such a distinction made in previous literature, see, e.g., (Lefkowitz, 1966; Mesarovic, et al., 1970; Findeisen et al., 1980). It was the development of the computer technology that enabled the realization of more computationally demanding advanced control algorithms based on process models, such as the popular DMC algorithm and other MPC type algorithms, leading to a separation of a dedicated supervisory feedback control layer. Since that time this distinction is commonly met in the papers of leading vendors delivering control equipment and software, as well as in review papers and basic textbooks, especially those devoted to process control, see, e.g., (Qin and Badgwell, 2003; Marlin, 1995; Goodwin, et al., 2001; Blevins, et al., 2003; Tatjewski, 2007, 2008). It should also be mentioned that the set-point control layer can not always occur. It is not distinguished in cases when there is no need for the set-point feedback control (constraint control) in the sense described above. Moreover, this layer can be not fully separating the direct control layer from the optimization layer – as indicated in the structure of Fig. 1.

When the standard multilayer control system structure is discusses, it is assumed that the individual layers are clearly distinct in the sense that each higher layer operates with an intervention frequency significanthy lower (i.e., with sampling interval significantly longer) than the intervention frequency of the directly subordinated layer. E.g., in process control typical sampling intervals of the direct control layer can be in the range of seconds, whereas sampling interval of the constraint control layer can be in the range of minutes and the steady-state optimization can be activated every hour or even less frequent (Qin and Badgwell, 2003). This main assumption is justified if main disturbances influencing performance of control units or on-line optimization units of individual layers have dynamics sufficiently slow as compared to the sampling intervals of these units, i.e., are appropriately slowly-varying when compared to the dynamics of the controlled process. In particular, if the dynamics of external disturbances shifting economically optimal set-point is significantly slower than the dynamics of the underlying controlled plant, than activating the optimization significantly less frequent than the intervention frequency of the feedback control is reasonable, otherwise this may lead to economic losses. However, in many important cases, the dynamics of main disturbances is comparable with the process dynamics. Very often these disturbances, for example flow rates, properties of feed and energy streams etc., vary significantly and not much slower than the dynamics of the controlled process. Certainly, in the era of powerful computers the control system designers try to cope with that problem which leads to stronger interactions between layers and control structures emerge where the discussed clear distinctions between layers does not hold. These will be focus points of Sections 3 and 4 of this presentation.

The paper is organised as follows. The following Section 2 will be devoted to selected problems concerning advanced control algorithms, in fact exclusively MPC algorithms. The full, numerical realisations of these algorithms relying on on-line constrained optimization will be discussed, with the focus on effective ways to cope with process nonlinearity and uncertainty. In particular, the idea of predictive control algorithms with fast model selection will be presented. In Section 3 the on-line set-point optimization with close relation to or even integrated with MPC control layer will be discussed. Finally, in Section 4 an application of MPC technology to the construction of a supervisory unit manipulating the set-points for basic (unconstrained) controllers applying integrated set-point optimization and constraint handling will be presented.

2. MPC FOR ADVANCED CONTROL OF NONLINEAR AND UNCERTAIN PROCESSES

The dominant advanced control technology is now undoubtedly the MPC (Model Predictive Control), developed in the last decades, although advanced control algorithms are usually defined as all those more advanced than the classical PID ones, see, e.g., (Maciejowski, 2002; Blevins et al., 2003). The main reasons for the dominance of the MPC technology have been just stated in the Introduction – and due to this dominance the advanced control layer is even sometimes called *the MPC layer*. Therefore, only the MPC algorithms are considered at this control layer in this paper. However, rapid development of computer technology made it possible to apply now MPC also for direct control (in simpler versions if necessary), when improved control performance is required and cannot be achieved with PIDs. In general, MPC refers to a class of computer control algorithms which at every sampling instant compute a sequence (a trajectory) of increments of manipulated variables which minimize predicted control errors over a prediction horizon. A typical formulation of the MPC dynamic optimization (MPC-DO) problem based on the process description in the form of an input-output model (e.g., step responses or ARX type model) is as follows, assuming the case dim y = dim u:

$$\min_{\Delta \boldsymbol{u}(k)} \{J_{MPC}(k) = \sum_{p=1}^{N} \left\| y^{sp}(k+p|k) - y(k+p|k) \right\|^2 + \\ + \lambda \sum_{p=0}^{N_u-1} \left\| \Delta u(k+p|k) \right\|^2 \}$$
$$u_{\min} \le u(k+p|k) \le u_{\max}, \quad p = 0,\ldots,N_u - 1 \\ -\Delta u_{\max} \le \Delta u(k+p|k) \le \Delta u_{\max}, \quad p = 0,\ldots,N_u - 1 \\ y_{\min} \le y(k+p|k) \le y_{\max}, \quad p = 1,\ldots,N \tag{1}$$

where $\Delta \boldsymbol{u}(k) = [\,\Delta u(k|k)^T \;\ldots\; \Delta u(k+N_u-1|k)^T\,]^T$ is the vector of decision variables (controller outputs) of dimension n_uN_u, (n_u=dim$u$), $y(k+p|k)$ denotes the process output prediction for the future sampling instant $k+p$, calculated at the current sample k using the process model; N and N_u denote prediction and control horizons, respectively, and λ is a weighting coefficient (diagonal weighting matrices can be used, in general). The constant set-point trajectory is usually assumed over the prediction horizon in process control applications, i.e., $y^{sp}(k+p|k) = y^{sp}(k)$, p=1,...,N. Actually only the first element of $\Delta \boldsymbol{u}(k)$, $\Delta u(k) = \Delta u(k|k)$ is applied and the whole procedure is repeated at the next sample, etc. (receding horizon approach).

Crucial element of the MPC-DO formulation is *the process model*. When it is linear (affine), then vector of predictions $\boldsymbol{y}(k) = \left[y(k+1|k)^T \;\ldots\; y(k+N|k)^T\right]^T$ can be, due to the superposition principle, decomposed to the sum

$$\boldsymbol{y}(k) = \mathbf{G}\Delta \boldsymbol{u}(k) + \boldsymbol{y}^0(k) \\ = \begin{bmatrix} \mathbf{g}_1 \Delta \boldsymbol{u}(k) + y^0(k+1|k) \\ \mathbf{g}_2 \Delta \boldsymbol{u}(k) + y^0(k+2|k) \\ \vdots \\ \mathbf{g}_N \Delta \boldsymbol{u}(k) + y^0(k+N|k) \end{bmatrix} \tag{2}$$

where $\boldsymbol{y}^0(k) = \left[y^0(k+1|k)^T \;\ldots\; y^0(k+N|k)^T\right]^T$ is the "free trajectory" (or "free response") vector of the length n_yN (n_y =dim$y$) depending on the past only, whereas $\mathbf{G}$ is the dynamic matrix of dimension $n_yN \times n_uN_u$, composed of coefficients of process step responses. The $\mathbf{G}\Delta \boldsymbol{u}(k)$ component of (2), depending only on the future input increments $\Delta \boldsymbol{u}(k)$, is often called the "forced trajectory". The resulting MPC-DO is then a QP problem, thus can be easily implemented for on-line application at constraint control layer, and also at direct control layer if sufficient computing power and adequate software is available in the DCS controller used. If it is not the case and simpler implementation is required then inequality constraints can be neglected when solving the (then unconstrained) MPC-DO problem – then its solution can be derived analytically off-line, as a MPC explicit, unconstrained control law, depending on the form of the process model used. It can be also often efficiently used in the presence of process input constraints simply projecting the controller output on this constraints, if only certain implementation principles are preserved, see (Tatjewski, 2007, 2008). An alternative to this suboptimal approach, especially for smaller problems without significant uncertainties, can be a design and use of an explicit piecewise-affine (constrained) MPC control law (Bemporad, et al., 2002; Tondell, et al., 2003).

The application of MPC algorithms with linear process models for supervisory constraint control in late 1970's and 1980's was a breakthrough, opening a new era in process control. Better control performance at feedback control layers made it possible to increase production effectiveness based on on-line process optimization. However, on-line economical set-point optimization means changing (adopting) the set-points to varying disturbances and production requirements, thus shifting the set-points to different regions of the usually nonlinear process, corresponding to different locally linearized process models. Therefore, nonlinear feedback control was needed. This led to development of nonlinear MPC algorithms. Presentation of this broad area is beyond the scope of this presentation, see, e.g., (Mayne, et al., 2000). Let us only mention that the formulation of MPC-DO (1) remains in fact unchanged, only the predictions $y(k+p|k)$ are now based on a nonlinear model, thus leading to a nonlinear MPC-DO problem, often difficult to on-line implementation. A practical, suboptimal but in most cases very satisfactory approach is to base part of the prediction depending on the past (free trajectory) on a nonlinear model, and prediction depending on decision variables (forced trajectory) on a linear model stemming from on-line linearization of the nonlinear one – the formula (2) remains valid, but with nonlinear $\boldsymbol{y}^0(k)$ and $\mathbf{G} = \mathbf{G}(k)$ stemming from current linearization, see, e.g., (Tatjewski, 2007). The structure of this MPC-NPL (MPC with Nonlinear Prediction and Linearization) algorithm is presented in Fig. 2. Certainly, for weakly nonlinear processes the linearization needs not be made at every sampling instant, it can be performed every few samples, their number depending on the nonlinearity. On the other hand, for strongly nonlinear processes and large set-point or disturbance changes the standard NPL approach may be not sufficiently effective. Certain remedy for that may be the use of process linearization around the predicted nonlinear free trajectory, or even a repetitive approach with a few successive linearizations around the successively improving predicted trajectory, see (Tatjewski, 2007).

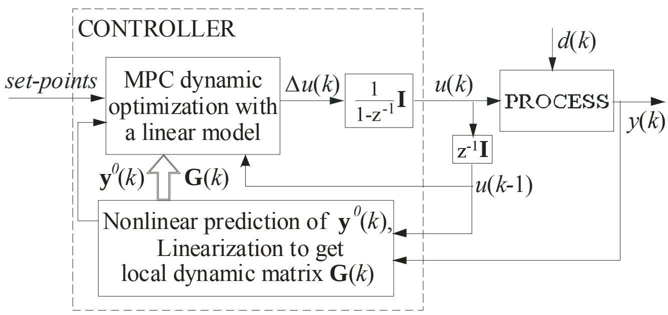

Fig. 2. Structure of the MPC-NPL algorithm

Another main challenge for the design of MPC algorithms are, however, the ways to cope with process uncertainties and changes. As in classical feedback control, we have two main approaches here:

- An *off-line approach*: design of a robust MPC algorithm taking into account a model of possible uncertainties at the design phase.
- An *on-line approach*: design of an adaptive MPC algorithm modifying on-line the control algorithm basing on current process measurements.

In the first case, the simplest practical approach is to detune the algorithm taking appropriately large values of the weighting coefficients for the part of the MPC cost function which penalises future control increments over the control horizon. This leads, however, to decreased control performance under nominal conditions and needs excessive simulations of the process behaviour under all possible operating conditions. The more elaborate approaches for the design of robust MPC controllers base on the "worst case principle". This leads to min-max approaches which are usually computationally prohibitive for on-line applications for most processes and are also known to lead to conservative designs, the larger the uncertainty the more conservative the controller. The worst-case min-max approach can be open-loop or closed-loop, the letter leads to a less conservative design, but needs even much more computations than the first one, thus is usually regarded as completely not practical. Let us remind that a complete description of the uncertainty is needed at the design phase of the robust controller, i.e., structure of the model uncertainty and ranges of its possible parameter variations.

In the second case, standard approach of indirect adaptive MPC is usually used, i.e., an on-line process identification procedure is applied and successive adaptation of the process model used in the MPC algorithm is performed, at every sampling instant or after every preselected number of sampling instants. The moving horizon estimation (MHE) is regarded here as a recommended one (Rao and Rawlings, 2002, 2005). The indirect adaptive approach has known advantages and drawbacks, the main drawback being the need to assure conditions of reliable on-line model identification, which can be difficult in feedback control loop.

A new, effective approach to uncertainty combining advantages of the two approaches described above will be now presented. It relies on complete description of the uncertainty but, unlike the robust approach, on-line modification of the model parameters is performed but, unlike the classical adaptive approach, the typical independent and involved model identification is not used – only a simple recalculation of the model parameters relying on the prediction procedure is performed. The approach will be called the MPC with fast on-line model selection – MPC-MS (Model Selection). It can be applied to any kind of a process parametric model, under certain assumptions on possible parameter values. It will now be presented for the MPC formulation with the exemplary state-space process model, assumed in the following form

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{3}$$

where the model matrices A and B are uncertain – can be varying with time, but are assumed to be always from the set being the convex hull of a given number, say L, of "vertex" models, i.e.,

$$(A(k), B(k)) \in co\{(A_1, B_1),...,(A_L, B_L)\}\,. \tag{4}$$

For further considerations, we will be using the following formulation of the MPC-DO problem with the assumed state-space process model, which is slightly more involved than (1) due to introduction of a terminal constraint set:

$$\min_{\{u(k+p|k)\}_{p=0}^{N-1}} \{J_{MPC}(k) = \sum_{p=1}^{N} \left\| x^{sp}(k+p) - x(k+p\,|\,k) \right\|_Q^2 + \sum_{p=0}^{N-1} \left\| u(k+p\,|\,k) \right\|_R^2 \} \tag{5}$$

$$u(k+p\,|\,k) \in U,\ p = 0,...,N-1$$
$$x(k+p+1\,|\,k) = A(k)x(k+p\,|\,k) + B(k)u(k+p\,|\,k) \in X,\quad p = 0,...,N-1$$
$$x(k+N\,|\,k) \in X_T$$

where $x(k|k) = x(k)$ denotes the current measured value of the state and X_T is a terminal set for the state applied to assure stability (a positive invariant set under a linear state feedback K_T). Further, for simplicity, the set-point equal to zero and full state measurement will be assumed.

The fast on-line model selection will be performed using the defined set of vertex models for state predictions. At sampling instant k, based on previous control input and previously measured state, $(u(k-1), x(k-1))$, current state predictions are calculated for every vertex model

$$x^j(k\,|\,k-1) = A_j x(k-1) + B_j u(k-1),\ j = 1,...,L \tag{6}$$

Then, for predictions in the MPC algorithm, the model is selected which minimizes the norm of a current prediction error

$$\left\{\hat{\lambda}_1(k),...,\hat{\lambda}_L(k)\right\} = \arg\min_{\{\lambda_j\}_{j=1}^{L}} \left\| x(k) - \sum_{j=1}^{L} \lambda_j x^j(k\,|\,k-1) \right\|,\quad \forall_{j=1,...,L}\ \lambda_j \in [0 \div 1],\ \sum_{j=1}^{L} \lambda_j = 1 \tag{7}$$

$$A(k) = \sum_{j=1}^{L} \hat{\lambda}_j(k) A_j,\ \ B(k) = \sum_{j=1}^{L} \hat{\lambda}_j(k) B_j \tag{8}$$

That is, prediction equations at sampling instant k are

$$x(k+p+1\,|\,k) = \sum_{j=1}^{L} (\hat{\lambda}_j(k) A_j)\, x(k+p\,|\,k) + \sum_{j=1}^{L} (\hat{\lambda}_j(k) B_j)\, u(k+p\,|\,k),\ \ p = 0,..., N-1 \tag{9}$$

Stability analysis of uncertain predictive control systems is generally a very difficult problem. For the considered MPC-MS algorithm stability results have been obtained (Sztyber, 2008), provided the algorithm is augmented with an additional safeguarding mechanism, based on the following condition:

$$\forall_{x \in X^p} \forall_{j=1 \div L} \{x_j^{p+1} = A_j x + B_j \hat{u}(k+p\,|\,k)\} = X^{p+1} \subset X,\quad X^N \subset X_T,\quad p = 0,...,N-1,\quad X^0 = \{x(k)\} \tag{10}$$

Theorem 1. Assume that there exist matrices F and $P > 0$ such that

1. sets X_T, X are convex,
2. $\forall_{x \in X} Fx \in U$,
3. $\forall_{x \in X} \forall_{j=1 \div L} (A_j + B_j F)x \in X$,
4. $\forall_{j=1 \div L} ((A_j + B_j F)^T P (A_j + B_j F) - P) < 0,$

then the following algorithm: "if condition (10) is satisfied then $\hat{u}(t \mid t)$ generated by MPC-MS is applied to the process, otherwise the process input is set to $u_f = Fx$" is stable.

EXAMPLE (Sztyber,2008). The second order process is simulated, described by matrices from the uncertainty set

$$(A,B) \in co \left\{ \begin{array}{c} (A_1 = \begin{bmatrix} 0 & 1 \\ -1.6 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}), \\ (A_2 = \begin{bmatrix} 0 & 1 \\ 0.1 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}) \end{array} \right\}. \tag{11}$$

MPC controller was designed for the process matrices (A_m, B_m) lying in the middle of the above set,

$$A_m = \begin{bmatrix} 0 & 1 \\ -0.7 & 1 \end{bmatrix}, \; B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{12}$$

The MPC-MS controller was implemented with

$$\begin{aligned} & N = 3, \; Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; R = [1], \\ & K_T = [0.7492 \;\; -0.9890], \\ & X_T = \left\{ x : x^T \begin{bmatrix} 19.5639 & -0.2347 \\ -0.2347 & 21.2817 \end{bmatrix} x \le 5 \right\} \end{aligned} \tag{13}$$

During the simulation the process was assumed to be (A_1, B_1), the constraints were $U = [-1 \div 1]$, $X = \Re^{n_x}$.

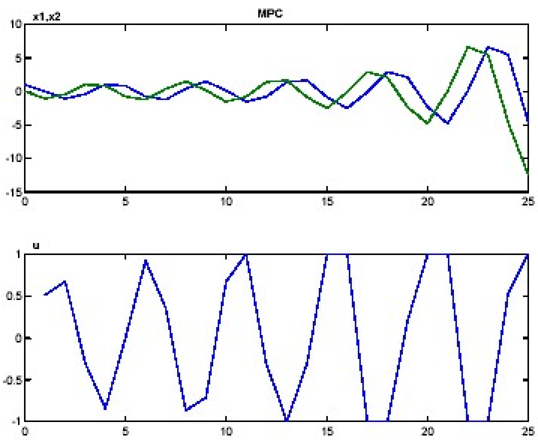

Fig. 3. Nominal MPC: state variables (upper plot) and the control signal (lower plot).

Performance of nominal MPC and MPC-MS algorithm is presented in Figures 3 and 4 showing state variables, control signal and optimal model parameters (7) as functions of time.

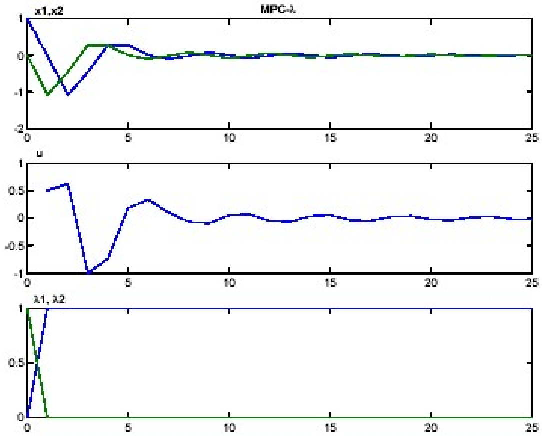

Fig. 4. MPC-MS: state variables (upper plot), control signal (middle plot) and optimal model parameters (lower plot).

3. ON-LINE SET-POINT OPTIMIZATION FOR ADVANCED FEEDBACK CONTROL

Formulation of the economic set-point optimization at the optimization layer of the control structure depends, in general, on dynamical properties of disturbances, i.e., process inputs not controlled at the optimization layer (coming as "true" disturbances from the process environment or as demands from higher layers of the hierarchy). Two general cases can be distinguished here:

1. Dynamic trajectories of disturbance predictions over certain time horizon are available, e.g., water demands in water treatment and distribution systems, prescribed dynamic trajectories of certain variables in batch processes, etc.
2. Measured or estimated current values of disturbances are only available.

The first case leads to dynamic set-point optimization, the second applies mainly to continuous control, leading usually to steady-state set-point optimization.There are two main cases here:

2.1. Disturbances are slow-varying when compared to the controlled process dynamics or changing abruptly but rare, (e.g., when switching to a different source of feed flows or changing the product specifications).

2.2. Disturbance dynamics is comparable with feedback controlled process dynamics.

Assuming linear form of the economic performance function, the model-based local steady-state optimization problem (LSSO) can be stated as the following LP problem

$$\begin{aligned} & \min_{u^{ss}, y^{ss}} \{ J_E(k) = c_u^T u^{ss} - c_y^T y^{ss} \} \\ & u_{\min} \le u^{ss} \le u_{\max} \\ & y_{\min} \le y^{ss} \le y_{\max} \\ & y^{ss} = F(u^{ss}, \tilde{w}) \end{aligned} \tag{14}$$

where $F(u,w)$ denotes a comprehensive steady-state process model, usually a nonlinear mapping, often given in an implicit numerical form, $\tilde{w}$ is the current estimate or measurement of disturbances, c_u and c_y are prices resulting from economic considerations, $u_{\min}$, $u_{\max}$, $y_{\min}$, $y_{\max}$ are constraint limits imposed on process input and output variables, respectively. Further, n_u, n_w, n_y denote numbers of process input variables u, disturbances w affecting the plant and output variables y, respectively. Let us point out that updating the model is a key issue at the optimization layer. It is a difficult problem for nonlinear constrained dynamic models, a popular technique is the extended Kalman filtering, recently the MHE is recommend. For steady-state models the situation is known to be simpler, for faster changing disturbances the key issue may be the time needed for relevant model update and re-optimization, as it may limit the frequency of optimization execution. The issues of model identification (update) are beyond the scope of this paper.

In case 2.1, there are long time intervals when disturbance values can be treated as constant parameters. Then the classical multilayer structure from Fig. 1 applies without loss of economic efficiency, with the optimization layer performing a steady-state (static) optimization rarely, much less frequent than the advanced controller executes. When a significant inaccuracy in the usually complex, comprehensive nonlinear steady-state process model is a problem, then in this case certain iterative algorithms based on steady-state feedback may lead to improvement (Roberts, 1979; Brdyś and Tatjewski, 2005; Tatjewski, 2007).

In case 2.2 dynamics of disturbances is comparable with the controlled process dynamics. In this case operation in the classical hierarchical structure with frequency of the economic optimization much lower than that of MPC can result in a significant loss of economic effectiveness – as keeping then the set-point constant over many sampling periods waiting for the next calculation of LSSO despite changes in disturbances, could lead to economic losses. Certainly, the most obvious and successful approach would be to perform the LSSO as often as needed, even as frequent as the MPC controller executes. However, the LSSO uses a comprehensive, nonlinear steady-state model of the process, performing its identification and nonlinear constrained optimization. This may be a difficult and time-consuming task, not possible to be executed on-line at each or even at every few sampling periods of the MPC.

Therefore, a simpler approach became an industrial practice: the use of an additional *steady-state target optimization* (SSTO) coupled with the MPC algorithm, see, e.g., (Kassmann et al., 2000; Blevins et al., 2003; Qin and Badgwell, 2003; Tatjewski, 2007}. The resulting control structure is depicted in Fig. 3, where direct interconnection of the optimization unit with the direct control layer is omitted (compare Fig. 1), as this simplifies the presentation, without loss of generality. As usual in a multilayer structure, each control unit (functional control block) calculates its output with a different frequency, the higher the block in the structure the lower the frequency – but MPC SSTO and MPC dynamic optimization (MPC-DO) constitute now functionally one control unit operating with the same sampling period.

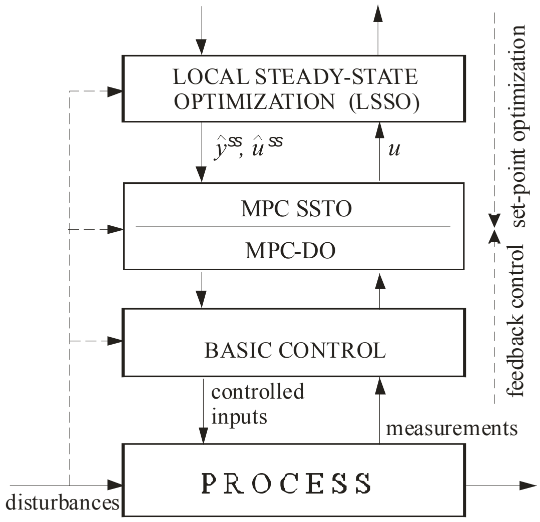

Fig. 5. Control structure with SSTO

Denote by $\hat{y}^{ss}, \hat{u}^{ss}$ steady-states calculated at the LSSO layer and transmitted to the lower layer, as in in Fig. 5. The role of the SSTO is to recalculate these values every time the MPC controller executes, to cope with varying disturbances. Denote these recalculated values by y^{ss}, u^{ss} (without "hats"). There are two main approaches to the SSTO: in the first the goal is to follow the set-points $\hat{y}^{ss}, \hat{u}^{ss}$ by the targets y^{ss}, u^{ss} as close as possible (Rao and Rawlings, 1999); in the second the set-points are optimally recalculated basing on the original economic performance function (that from LSSO), but using a simplified steady-state process model (Kassman, et al., 2000; Blevins, et al., 2003). The second approach is more important and interesting, as the SSTO is then a simplified version of the LSSO problem (14). In many cases a steady-state version, i.e., the gains matrix, of the linear dynamic model used in the MPC-DO is reported to be used (Kassmann, et al., 2000; Blevins et al., 2003; Qin and badgwell, 2003). This results in a linear programming (LP) problem, if the economic objective function is linear. Having LSSO in the form (14), the SSTO takes then the form

$$\begin{aligned} &\min_{u^{ss},y^{ss}} \{J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss}\} \\ &\Delta y^{ss} = \mathbf{G}_{ss} \Delta u^{ss} \\ &y^{ss} = y^0(k+N\mid k) + \Delta y^{ss} \\ &u^{ss} = u(k-1) + \Delta u^{ss} \\ &u_{\min} \le u^{ss} \le u_{\max} \\ &y_{\min} \le y^{ss} \le y_{\max} \end{aligned} \tag{15}$$

where $\mathbf{G}_{ss}$ is the gains matrix of the dynamic model used in MPC-DO. The LP problem (15) is usually solved at each sampling instant of the MPC algorithm, after prediction of the free output $y^0(k+N|k)$, before solution of the MPC-DO problem.

However, the constant linear model used in (15) may be too different from the nonlinear one used in the LSSO, for most of the operating points, which may lead to losses in economic optimality. Hence, Qin and Badgwell (2003) report on using linearizations of the nonlinear model instead of the gain matrix of the dynamic one. The reason is that he model used in SSTO should be consistent with the comprehensive steady-state nonlinear model from the LSSO layer, rather than with the dynamic one applied in the MPC. Using the linearization of the LSSO model $F(\cdot,\widetilde{w}(k))$, at sampling instant k (*i.e.*, at the point $u(k$-1)), leads to the adptive SSTO problem of the following LP form

$$\begin{aligned}
&\min_{u^{ss},y^{ss}}\{J_E(k)=c_u^T\Delta u^{ss}-c_y^T\Delta y^{ss}\}\\
&\Delta y^{ss}=\mathbf{H}(k)\Delta u^{ss}\\
&y^{ss}=F(u(k-1),\widetilde{w}(k))+\Delta y^{ss}\\
&u^{ss}=u(k-1)+\Delta u^{ss}\\
&u_{\min}\le u^{ss}\le u_{\max}\\
&y_{\min}\le y^{ss}\le y_{\max}
\end{aligned} \tag{16}$$

which is similar to (15), but with $F(u(k-1),\widetilde{w}(k))$ instead of $y^0(k+N|k)$ and with $\mathbf{H}$(k) in place of $\mathbf{G}_{ss}$. The SSTO problem (16) should be solved at each sampling instant, but it may be reasonable to update the nonlinear model $F(\cdot,\widetilde{w}(k))$ and the gains matrix $\mathbf{H}$(k) not so frequent. Updating $\mathbf{H}$(k) is sensible after a significant change in the model nonlinearity, similarly the value $F(u(k-1),\widetilde{w}(k))$ after a reasonably significant change in disturbance measurement/estimate $\widetilde{w}$. The resulting control structure is presented in Fig. 6, see also (Tatjewski, 2007). The simulations have shown that this structure leads in most cases to the control performance very close to the reference (and not realistic) one from Fig. 1 with LSSO repeated at every sample of the MPC (Ławryńczuk et al., 2007b).

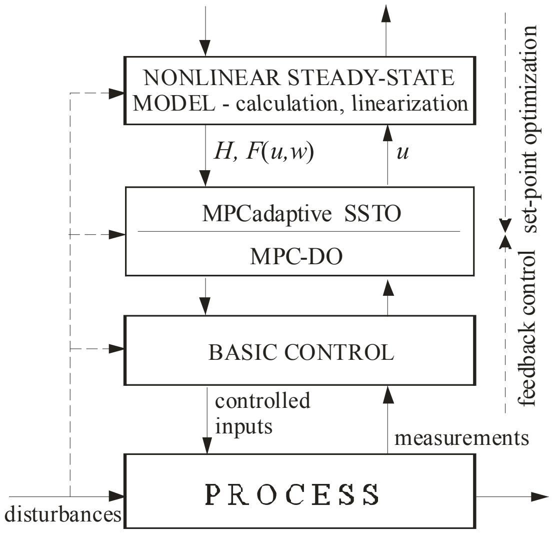

Fig. 6. Control structure with adaptive SSTO

Because optimization problems (15) and (16) are solved at the same sampling instants, it is possible to integrate them into one problem, e.g. adding appropriately weighted cost function from (16) to the cost function of (1) and summing the constraints, this leads to the following integrated MPC-DO-SSTO problem:

$$\min_{\Delta\boldsymbol{u}(k),u^{ss},y^{ss}}\{J_{INT}(k)=\sum_{p=1}^{N}\left\|y^{ss}-\mathbf{g}_p\Delta\boldsymbol{u}(k)-y^0(k+p\,|\,k)\right\|^2+$$
$$+\lambda\sum_{p=0}^{N_u-1}\left\|\Delta u(k+p\,|\,k)\right\|^2+\gamma(c_u^Tu^{ss}-c_y^Ty^{ss})\}$$

subject to :

$$\begin{aligned}
&u_{\min}\le u(k-1)+\sum_{i=0}^{p}\Delta u(k+i\,|\,k)\le u_{\max}, && p=0,\dots,N_u-1\\
&-\Delta u_{\max}\le\Delta u(k+p\,|\,k)\le\Delta u_{\max}, && p=0,\dots,N_u-1\\
&y_{\min}\le\mathbf{g}_p\Delta\boldsymbol{u}(k)+y^0(k+p\,|\,k)\le y_{\max}, && p=1,\dots,N\\
&u_{\min}\le u^{ss}\le u_{\max}\\
&y_{\min}\le y^{ss}\le y_{\max}\\
&y^{ss}=F\big(u(k-1),\widetilde{w}\big)+\mathbf{H}(k)\big(u^{ss}-u(k-1)\big)
\end{aligned} \tag{17}$$

where γ is a weighting coefficient. The approach was investigated by simulation for several process control models, in many cases it leads to acceptable results very close to those from the multilayer approach from Fig. 6 (Ławryńczuk et al., 2007a, 2007b). Generally, the integration is not a novel idea, there were earlier attempts to integrate nonlinear LSSO with MPC-DO (without linearizations), for a specific process (Zanin et al., 2002).

It should be noticed that further development of the SSTO problem, for strongly nonlinear processes, has been also proposed (Tatjewski, et al., 2006; Tatjewski, 2007) involving quadratic and piecewise-linear approximations.

It should be pointed out that for successful nonlinear MPC control the form, the choice of the nonlinear process model is of great importance. It is not only vital when a nonlinear MPC-DO problem is solved on-line, but also when the nonlinear model must be frequently updated and linearized, as in the structures discussed in this section. The use of computationally good fuzzy and neural net models, being often approximations of involved original nonlinear first-principle models, is here recommended (Tatjewski, P., and M. Ławryńczuk, 2006; Tatjewski, 2007). An application of a Hammerstein model with fuzzy nonlinear part is described in a parallel paper presented at this workshop (Ławryńczuk, at al., 2009b).

4.TWO-PURPOSE PREDICTIVE SET-POINT OPTIMIZER FOR UNCONSTRAINED BASIC CONTROLLERS

There is a class of processes having a basic feedback control layer equipped with unconstrained controllers (usually PIDs), for which a supervisory advanced feedback control layer is not needed – but improved economic set-point optimization and, first of all, handling of process input and output constraints is required. In (Ławryńczuk, et al., 2009a) a control structure was described in which a supervisory predictive model-based economic optimiser is responsible not only for on-line set-point optimization but also for fulfilment of the constraints, it is presented in Fig. 7. The optimizer acts as a set-point governor for satisfaction of constraints in the underlying basic feedback controlled system. The idea of reference value (set-point) governors is not new, see, e.g., (Bemporad, A., 1998). On the other hand in (Saez, D., et al., 2002) design of a similar predictive set-point optimiser, for generally unconstrained processes, was presented.

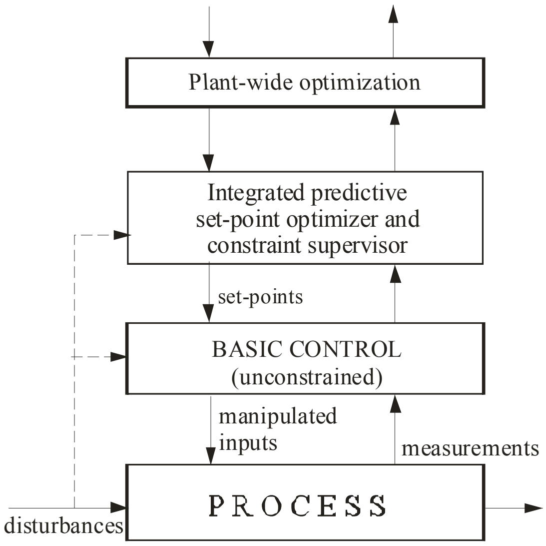

Fig. 7. Control structure with integrated set-point supervisor

The supervisor described here is constructed by integration of three elements: the MPC technique, the economic optimization and models of the basic unconstrained controllers. The integrated problem involving the optimization with nonlinear process model (14) is as follows:

$$\begin{aligned}
&\min_{u^{ss},y^{ss},\Delta\boldsymbol{u}(k)} \{J_{INT}(k) = \sum_{p=1}^{N} \left\| y^{ss} - \mathbf{g}_p \Delta\boldsymbol{u}(k) - y^0(k+p\,|\,k) \right\|^2 + \\
&\qquad + \lambda \sum_{p=0}^{N_u-1} \left\| \Delta u(k+p\,|\,k) \right\|^2 + \gamma(c_u^T u^{ss} - c_y^T y^{ss})\} \\
&u_{\min} \le u(k-1) + \sum_{i=0}^{p} \Delta u(k+i\,|\,k) \le u_{\max}, \quad p = 0,\ldots,N_u-1 \\
&-\Delta u_{\max} \le \Delta u(k+p\,|\,k) \le \Delta u_{\max}, \quad p = 0,\ldots,N_u-1 \\
&y_{\min} \le \mathbf{g}_p \Delta\boldsymbol{u}(k) + y^0(k+p\,|\,k) \le y_{\max}, \quad p = 1,\ldots,N \\
&u_{\min} \le u^{ss} \le u_{\max} \\
&y_{\min} \le y^{ss} \le y_{\max} \\
&y^{ss} = F\left(u^{ss}, \tilde{w}\right) \\
&\Delta\boldsymbol{u}(k) = \boldsymbol{K}(\boldsymbol{y}^{ss} - \boldsymbol{y}^0(k))
\end{aligned} \tag{18}$$

where $\boldsymbol{y}^{ss} = \left[y^{ss\,T} \;\ldots\; y^{ss\,T} \right]^T$ is the vector consisting of "repeated" y^{ss} to be consistent with $\boldsymbol{y}^0(k)$ as defined directly after (2). In (18) $\boldsymbol{K}$ describes a linear internal feedback control law applied for basic control, which can be, e.g., the unconstrained MPC control law with $\boldsymbol{K} = (\boldsymbol{G}^T\boldsymbol{G} + \boldsymbol{\Lambda})^{-1}$ as it is in (18), or the PID law, $\boldsymbol{u}(k) = \boldsymbol{R}(\boldsymbol{y}^{ss} - \boldsymbol{y}(k))$. Observe that due to the internal feedback controller model included into (18), the control vector $\Delta\boldsymbol{u}(k)$ is in fact *no longer an independent variable,* it depends linearly on the control error vector. In fact, the only independent variable is the input set-point u^{ss}, as y^{ss} results from the steady-state process model.

The problem (18) is a nonlinear one. In order to transform it into a quadratic programming problem, more realistic to be solved on-line, a natural approach is to linearize the steady–state process model, taking into account the current state of the plant. As a result the following optimization problem is obtained, comp. with (18) and (17):

$$\begin{aligned}
&\min_{u^{ss},y^{ss},\Delta\boldsymbol{u}(k)} \{J_{INT}(k) = \sum_{p=1}^{N} \left\| y^{ss} - \mathbf{g}_p \Delta\boldsymbol{u}(k) - y^0(k+p\,|\,k) \right\|^2 + \\
&\qquad + \lambda \sum_{p=0}^{N_u-1} \left\| \Delta u(k+p\,|\,k) \right\|^2 + \gamma(c_u^T u^{ss} - c_y^T y^{ss})\} \\
&u_{\min} \le u(k-1) + \sum_{i=0}^{p} \Delta u(k+i\,|\,k) \le u_{\max}, \quad p = 0,\ldots,N_u-1 \\
&-\Delta u_{\max} \le \Delta u(k+p\,|\,k) \le \Delta u_{\max}, \quad p = 0,\ldots,N_u-1 \\
&y_{\min} \le \mathbf{g}_p \Delta\boldsymbol{u}(k) + y^0(k+p\,|\,k) \le y_{\max}, \quad p = 1,\ldots,N \\
&u_{\min} \le u^{ss} \le u_{\max} \\
&y_{\min} \le y^{ss} \le y_{\max} \\
&y^{ss} = F\left(u(k-1), \tilde{w}\right) + \mathbf{H}(k)\left(u^{ss} - u(k-1)\right) \\
&\Delta\boldsymbol{u}(k) = \boldsymbol{K}(\boldsymbol{y}^{ss} - \boldsymbol{y}^0(k))
\end{aligned} \tag{19}$$

Observe that also the output constraints can be taken into account in the supervisor action. After solving (14), the new set-point values y^{ss} are obtained Because the behaviour of the basic feedback controllers is taken into consideration by the set–point optimiser, not only process output $y(k)$ but also process input $u(k)$ generated by the internal controller are predicted and considered during calculation of the set-point values, which is performed taking into account the constraints put on predicted process output and input values. It should be mentioned that an alternative formulation of the problem (18) and (19) is possible: namely, the dynamic plant model and the local feedback controller can be described using one dynamic model of the closed loop. Such an approach results if the plant and the feedback controllers are modelled (identified) together.

EXAMPLE. The CSTR with van de Vusse reaction is considered (Doyle et al. 1995), shown in Fig. 8.

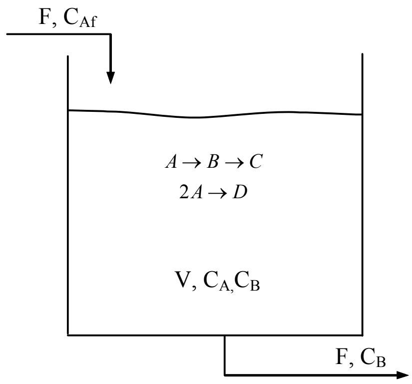

Fig. 7. CSTR with van de Vusse reaction

The state equations are as follows

$$\frac{dC_A}{dt} = -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A)$$
$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - \frac{F}{V} C_B \tag{20}$$

where C_A, C_B are compositions of components A and B, F is the feed inflow, V is the volume (assumed constant, V=1 l), C_{Af} is the composition of the component A in the feed stream and is a measured disturbance. Kinetic parameters of the reaction are: k_1=50 1/h, k_2=100 1/h, k_3=10 $l/h{\cdot}mol$.

The composition C_B of the product B in the output stream is the output variable, F is the manipulated input. It was assumed during the experiments that the disturbing composition C_{Af} varies according to the formula

$$C_{Af}(t) = C_{Af0} - \sin\left(\frac{2\pi}{100}t\right) \tag{21}$$

where C_{Af0}=10 mol/l. The economic performance function, yielding maximization of the production output, is

$$J_E = -F\,. \tag{22}$$

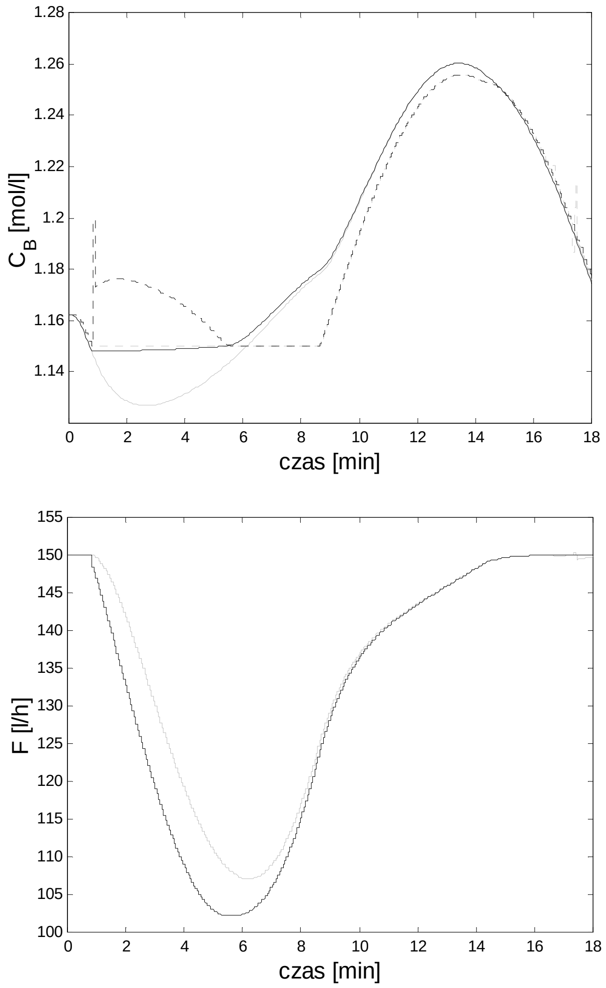

Fig. 8. Trajectories of the output and control signal in the multilayer structure with the integrated predictive supervisor (black line) and in the classical multilayer structure (grey line); dotted lines – set-point trajectories

The constraints are: 0 $l/h \le F \le$ 150 l/h for the process input and $C_B \ge$ 1,15 mol/l for the process output. They are taken into consideration both at the optimization layer and in the integrated predictive optimizer which generates set-points for the internal unconstrained controller. The latter is the analytic (unconstrained) DMC control law. The sampling time is T_p=3,6 s, and the parameter values: N=30, N_u=15, λ=0,001 were assumed when designing the DMC control law.

Fig. 8 presents results of simulation experiments in the presented multilayer control structure with integrated predictive supervisor and in the classical multilayer structure (basic feedback control + LSSO) . There is a clear difference between these cases, especially during the first phase of the simulation experiments, when difference between generated set-point trajectories for basic controllers is significant. This is due to the presence in the optimizer both the process and basic (internal) controller models and all the constraints.

5. CONCLUSIONS

The paper presented a short review and, first of all, selected topics of supervisory advanced control and on-line set-point optimization. It was indicated that for advanced MPC technology issues of computationally effective nonlinear and robust control are a challenge. Calculation of free output trajectory using a nonlinear model combined with the linearized model for optimization of control increments (thus leading to QP) was indicated as an effective practical way to cope with process nonlinearity. The new technique of MPC with fast on-line model selection was presented as a possible way to cope effectively with model uncertainty, in particular with models undergoing unknown, unpredicted changes, but within an a priori known parameter set. The importance of proper selection of the control structure for on-line set-point optimization for MPC for frequently met cases when dynamics of disturbances is comparable with the controlled process dynamics was pointed out. Control structures for this situation were discussed, starting from classical SSTO (steady-state target optimization) and presenting adaptive SSTO and an integrated approach. Finally, two-purpose integrated predictive set-point optimizer-governor was presented for cases where there is no need for supervisory advanced feedback control over unconstrained basic feedback control, but improved set-point optimization and constraint handling is required. A conclusion from the presented solutions is that a number of solutions emerged for the most difficult case with dynamics of disturbances comparable with controlled process dynamics, leading to non-conventional multilayer structures and algorithms. This corresponds to continuing development of control potential and possibilities of continuously developing control architectures for process automation (Samad, et. al., 2007).

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